Calculation of absorption coefficient of micro-perforated absorbers

This is a program calculating absorption coefficient of micro-perforated panels and membranes mounted over an airtight cavity. Using the program a required absorber can be designed, and the effects of various parameters can be demonstrated. For comparison, two configurations, one with a single layer and one with double layers, can be considered. Sound incident angle can also be specified.

Micro-perforated absorber is a newly developed sound absorber, and it has a number of attractive features:
- Unlike conventional perforated absorbers, micro-perforated absorber can be made from transparent materials like plastic glass.
- Unlike commonly used fibrous absorptive materials, micro-perforated absorber is fibrous-free and thus, there is no health concern.
- Micro-perforated membranes are lightweight and inexpensive.

The absorption performance of micro-perforated absorber is very good. Typically the absorption coefficient exceeds 0.4 over 3-5 octaves.

The program allows the following inputs:
1. Panel thickness
2. Aperture diameter
3. Aperture spacing
4. Air-space depth
5. Material (PVC or Metal)
6. Angle of sound incidence
7. Acoustic resistance of the membrane (normally 1 can be used)
8. Surface density of the membrane
Algorithm of the program

For more information see:

Theories for both the membrane absorber [1,2], namely a limp lightweight membrane backed by an air space, and the micro-perforated absorber [3,4], namely a cavity backed plate with low aperture ratio but many apertures of sub-millimetre size, have been well developed. However, it appears that no theory has been given for their combination, such as a glass-fibre textile or a micro-perforated membrane mounted over an airtight cavity. Measurements have shown that such a structure could act as a good absorber [5-7]. This absorber would have notable advantages in practice since it is lightweight, inexpensive, and due to the absence of loose fibrous material, less of a heath concern than commonly used fibrous absorptive materials. In this paper, a theoretical method for predicting the absorption of this structure is described, and the predictions are compared with measurements. A series of parameter studies is also presented.

SINGLE LAYER

For an open weave textile or a micro-perforated membrane backed by an air space, since the resonance frequencies caused by the membrane and by the apertures can be in the same range, their simultaneous effects should be considered. The basic idea of the proposed theory is to regard an open weave textile or a micro-perforated membrane as a parallel connection of the membrane and apertures. In this way the effectiveness of one element relies on its relative impedance to the other. In other words, if the acoustic impedance of the apertures is much greater than that of the membrane, the absorption of the structure depends mainly on the characteristics of the membrane, and conversely, the apertures will play a dominant role if their acoustic impedance is much less than that of the membrane. In order to modify the absorption, the acoustic impedance of the structure can be adjusted by varying the appropriate parameters of the membrane and the apertures.

An open weave textile or a micro-perforated membrane backed by an air space makes a resonance system. The acoustic impedance of such a system can be obtained using the impedance type of electro-acoustic analogy. Basically, the resonance system contains the mass-resistance element in series with the cavity reactance of the air space. As mentioned above, the mass-resistance element consists of the membrane and the apertures being connected in parallel. In Figure 1 is shown the equivalent circuit of a single layer of open weave textile or micro-perforated membrane mounted at distance $D$ (m) from a rigid wall, where $R_M$ and $M_M$ are the specific acoustic resistance and reactance of the membrane, and $R_L$ and $M_L$ are the specific acoustic resistance and reactance of the apertures. The sound wave impinging on the structure is equivalent to a source of sound pressure $2p$ as produced on the rigid wall (analogous to the open-circuit voltage) and internal resistance $\rho c$ as that of air [4].
From Figure 1 it can be seen that in order to obtain the acoustic impedance of the whole system, it is necessary to consider the impedance of each element. Consider first an unperforated membrane. For a tension-free membrane mounted at some distance from a rigid wall, the normal specific acoustic impedance of the membrane normalised by \( \rho c \) can be calculated by

\[
\frac{z_M}{\rho c} = R_M + j\omega M_M = r' + j\omega m''
\]

where \( \rho \) is the density of air and \( c \) is the sound velocity in air. \( \omega = 2\pi f \), \( f \) is the frequency (Hz). \( m'' = m'/\rho c, m' \) is the surface density of the membrane (kg/m²). \( r' \) is the normalised specific acoustic resistance of the membrane, which depends mainly on mounting conditions.

Consider secondly the effect of the apertures alone. An aperture may be regarded as a short tube. The propagation of sound waves in narrow tubes was first discussed by Rayleigh [8], and a simplified version was given by Crandall [9] for very short tubes in comparison with wavelengths. For the equation of aerial motion inside the tube, by assuming sinusoidal functions of time and zero velocity on the tube wall, an exact solution for the acoustic impedance of the tube was derived. Because the calculation was rather complicated, Crandall proposed two approximate formulae for both small and large apertures that can be used for porous materials and conventional perforated plates respectively. Maa [3,4], observing the discontinuity between the two cases, developed an approximate solution for apertures of sub-millimetre size, namely for micro-perforated plates. A micro-perforated plate backed by an air space can still be regarded as a mass-spring system. In comparison with conventional formulae for perforated plate absorbers, however, an outstanding feature of Maa's theory is that the acoustic resistance of the apertures, which becomes significant
when the apertures are very small, is taken into account. Consequently, for micro-
perforated absorbers it is not necessary to provide extra acoustic resistance using
porous materials. According to Maa, for normal incidence, the normalised specific
acoustic impedance of the apertures can be calculated by

\[ z_L = \frac{R_L + jM_L}{\rho c} = r + j\omega m \]  \hspace{1cm} (2)

with

\[ r = \frac{g_1 t}{d^2 p} K_r \]  \hspace{1cm} (3)

\[ m = 0.294(10^{-3}) \frac{t}{p} K_m \]  \hspace{1cm} (4)

\[ K_r = \sqrt{1 + \frac{x^2}{32} + \frac{x\sqrt{2}}{8} \frac{d}{t}} \]  \hspace{1cm} (5)

\[ K_m = 1 + \frac{1}{9 + \frac{x^2}{2}} + 0.85 \frac{d}{t} \]  \hspace{1cm} (6)

\[ x = g_2 d \sqrt{f} \]  \hspace{1cm} (7)

where \( t \) is the membrane thickness (mm), \( d \) is the aperture diameter (mm), \( p \) is the
aperture ratio (aperture area / membrane area), and \( b \) is the distance between aperture
centres (mm). \( g_1 \) and \( g_2 \) are constants. For non-metallic material, \( g_1 = 0.147 \) and
\( g_2 = 0.316 \). For metallic material, \( g_1 = 0.335 \) and \( g_2 = 0.21 \). The above equations are
used for circular apertures. For square apertures with a side \( l \) (mm), an
approximation can be made by using the same aperture area [5], namely \( d = 2l/\sqrt{1/\pi} \).
In equation (5) the second term is the end correction for resistance. Similarly, the last
term in equation (6) is the end correction for mass reactance. It is noted that
equations (3) to (7) are restricted to air under standard conditions of temperature and
pressure.

The normal specific acoustic impedance of the air behind the membrane, again
normalised by \( \rho c \), is

\[ z_D = -j\text{ctg}(\frac{\omega D}{c}) \]  \hspace{1cm} (8)

According to the equivalent circuit in Figure 1, the normalised normal specific
acoustic impedance of the whole structure can be calculated by

\[ z = \frac{z_M z_L}{z_M + z_L} + z_D = H_r + j(H_m - \text{ctg} \frac{\omega D}{c}) \]  \hspace{1cm} (9)
with
\[ H_r = \frac{H_a H_c + H_b H_d}{H_c^2 + H_d^2} \]  \hspace{1cm} (10)
\[ H_m = \frac{H_a H_c - H_b H_d}{H_c^2 + H_d^2} \]  \hspace{1cm} (11)
\[ H_a = rr' - \omega^2 \text{mm}'' \]  \hspace{1cm} (12)
\[ H_b = r' \omega m + r \omega m'' \]  \hspace{1cm} (13)
\[ H_c = r + r' \]  \hspace{1cm} (14)
\[ H_d = \omega(m + m'') \]  \hspace{1cm} (15)

The absorption coefficient can then be calculated by the well-known formula:
\[ \alpha = \frac{4 \text{Re}(z)}{[1 + \text{Re}(z)]^2 + [\text{Im}(z)]^2} = \frac{4H_r}{(1 + H_r)^2 + (H_m - \text{ctg} \frac{\omega D}{c})^2} \]  \hspace{1cm} (16)

The maximum absorption coefficient is
\[ \alpha_{\text{max}} = \frac{4H_r}{(1 + H_r)^2} \]  \hspace{1cm} (17)

As with other resonant absorbers, there are multiple resonance frequencies for the above structure, which can be calculated by using
\[ H_m - \text{ctg} \frac{\omega D}{c} = 0 \]  \hspace{1cm} (18)

Between the multiple resonances there are zero points in absorption, which occur at the frequencies making \( \text{ctg} (\omega D / c) \) infinite.

**OBLIQUE AND RANDOM INCIDENCE**

The membrane itself and the apertures are locally reacting acoustic elements [2-4]. That is, the acoustic impedance is independent of the angle of incidence. Conversely, the impedance of the air space will change due to the change of the path difference between the incident and reflected waves. Overall, when the sound wave is incident under an angle \( \theta \) to the normal, equation (9) becomes

\[ z_\theta = H_r \cos \theta + j(H_m \cos \theta - \frac{1}{\cos \theta} \text{ctg} \frac{\omega D \cos \theta}{c}) \]  \hspace{1cm} (19)
Using equation (17) it can be demonstrated that in comparison with normal incidence the maximum absorption coefficient for oblique incidence is less if $H_r <-1$, and can be greater when $H_r >1$. It can also be proved that for oblique incidence the absorption range will shift to higher frequencies [3].

In a diffuse sound field, the angle-averaged absorption coefficient can be obtained from

$$\alpha_0 = 2 \int_0^{\pi/2} \alpha_\theta \sin \theta \cos \theta d\theta$$

Clearly the zero points in absorption between the multiple resonances will disappear due to the angle average. In other words, the troughs in the absorption curve between the multiple resonances will be less deep in comparison with those under normal incidence condition. This is especially useful for wideband absorbers like open weave textiles or micro-perforated membranes. Similar to oblique incidence, in a diffuse field the resonance frequencies will be higher than those for normal incidence.

DOUBLE LAYER

To broaden the frequency range of absorption, it is effective to use more layers of membrane. In Figure 2 are shown a structure with two layers of open weave textile or micro-perforated membrane and the equivalent circuit of this structure, where $D_1$ (m) is the distance between the two layers and $D_2$ (m) is the distance from the inner layer to the rigid wall. In correspondence with the double resonator formed by the structure, the equivalent circuit has a second resonance circuit added parallel with $Z_{D_1}$. According to the equivalent circuit in Figure 2, the normalised normal specific acoustic impedance of the whole structure can be calculated by

$$z = H_{r1} + j(H_{m1} - \cot \frac{\omega D_1}{c}) + \frac{\cot \frac{\omega D_1}{c}}{H_{r2} + j(H_{m2} - \cot \frac{\omega D_1}{c} - \cot \frac{\omega D_2}{c})}$$

$$= H_{r1} + \frac{H_C H_{r2}}{H_B + H_{r2}^2} + j(H_A - \frac{H_B H_C}{H_B + H_{r2}^2})$$

with

$$H_A = H_{m1} - \cot \frac{\omega D_1}{c}$$

$$H_B = H_{m2} - \cot \frac{\omega D_1}{c} - \cot \frac{\omega D_2}{c}$$

$$H_C = \cot^2 \left( \frac{\omega D_1}{c} \right)$$

where $H_{r1}$ and $H_{m1}$ are calculated using the parameters of the outer membrane, and $H_{r2}$ and $H_{m2}$ are in correspondence with the inner membrane.
When only considering the effect of apertures, it has been theoretically demonstrated by Maa [3] that in comparison with a single layer micro-perforated plate with \( D_1 \), when there are two micro-perforated layers with \( D_1 + D_2 \), the low frequency range of absorption can be extended approximately from \( f_i \) to \( f_i [D_1 / (D_1 + D_2)] \). By using equation (21) it can be proved that this is also true when the effect of the membrane is taken into account. At relatively high frequencies, namely higher than the resonance frequency of the inner layer, by using the first expression of equation (21) it can be proved that the acoustic reactance of two layers is approximately the same as that of the outer layer alone. This means that using an additional layer may not increase the high frequency range of absorption. As for the acoustic resistance, from the second expression of equation (21) it is seen that using an extra layer can increase the real component of the acoustic impedance.

For a wave impinging on the structure at an angle \( \theta \) to the normal, in equations (21) to (24) \( H_{r1} \), \( H_{r2} \), \( H_{m1} \), \( H_{m2} \), \( D_1 \) and \( D_2 \) should be multiplied by \( \cos \theta \). Consequently, the diffuse field absorption coefficient can be calculated using equation (20).


